Variation of params: Find a particular solution for

$$y'' + 4y = \cos(9x) \leftarrow f(x) \neq 0$$

find y_c: $y'' + 4y = 0 \longrightarrow r^2 + 4 = 0$

$r = \pm 2i$ (mult. 1)

so general solution $y_c = c_1 \underbrace{\cos(2x)}_{y_1 = \cos(2x)} + c_2 \underbrace{\sin(2x)}_{y_2 = \sin(2x)}$

Variation of params is:

find u_1, u_2 (functions) such that

$$y = u_1 y_1 + u_2 y_2$$

method leads to

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = f = \cos(9x) \end{cases}$$

$$\begin{cases} u_1' \cos(2x) + u_2' \sin(2x) = 0 \\ u_2' \cdot (-2\sin(2x)) + u_2' \cdot (2\cos(2x)) = \cos(9x) \end{cases}$$

which (always) has solutions

$$\boxed{\begin{cases} u_1' = \dfrac{-y_2 \cdot f}{W(y_1, y_2)} \\ u_2' = \dfrac{+y_1 \cdot f}{W(y_1, y_2)} \end{cases}}$$

$$W(y_1, y_2) = \begin{vmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{vmatrix}$$

$$= 2\cos^2(2x) + 2\sin^2(2x)$$

$$= 2$$

$$u_1' = \frac{-\sin(2x)\cdot\cos(9x)}{2}$$

$$u_2' = \frac{\cos(2x)\,\cos(9x)}{2}$$

hint: trig identities

$$\sin(a)\cos(b) = \frac{1}{2}\left(\sin(a-b) + \sin(a+b)\right)$$

$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a-b) + \cos(a+b)\right)$$

$$u_1' = \frac{-\sin(2x)\cdot\cos(9x)}{2} = \frac{-1}{4}\left(\sin\left((2-9)x\right) + \sin\left((9+2)x\right)\right)$$

$$= \frac{-1}{4}\left(-\sin(7x) + \sin(11x)\right)$$

$$u_1 = \int u_1'\,dx = +\frac{1}{4}\left(-\frac{1}{7}\cos(7x) + \frac{1}{11}\cos(11x)\right) + C_1$$

$$u_2' = \frac{\cos(2x)\cos(9x)}{2} = \frac{1}{4}\left(\cos(7x) + \cos(11x)\right)$$

$$u_2 = \int u_2'\,dx = \frac{1}{4}\left(\frac{1}{7}\sin(7x) + \frac{1}{11}\sin(11x)\right) + C_2$$

$$u_1 y_1 + u_2 y_2 =$$

$$C_1\cos(2x) + C_2\sin(2x)$$

$$+ \frac{-1}{28}\cos(7x)\cos(2x) + \frac{1}{44}\cos(11x)\cos(2x)$$

$$+ \frac{1}{28}\sin(7x)\sin(2x) + \frac{1}{44}\sin(11x)\sin(2x)$$

hint: $\cos(a)\cos(b) + \sin(a)\sin(b) = \cos(a-b)$

$\cos(a)\cos(b) - \sin(a)\sin(b) = \cos(a+b)$

$$u_1 y_1 + u_2 y_2 = c_1 \cos(2x) + c_2 \sin(2x) - \frac{1}{28}\cos(9x) + \frac{1}{44}\cos(9x)$$

$$= \underbrace{c_1 \cos(2x) + c_2 \sin(2x)} \underbrace{- \frac{1}{77}\cos(9x)}$$

this is y_c, so this is y_p

Much easier way: Since $\cos(9x)$ is one of our "known cases", we can use Undetermined coeffs:

$$y'' + 4y = \underline{\cos(9x)}$$

$$y_c = c_1 \cos(2x) + c_2 \sin(2x)$$

$$y_p = A\cos(9x) + B\sin(9x)$$

$$y_p' = -9A\sin(9x) + 9B\cos(9x)$$

$$y_p'' = -81A\cos(9x) - 81B\sin(9x)$$

$$y_p'' + 4y_p = (4-81)A\cos(9x) + (4-81)B\sin(9x) = \cos(9x)$$

$$A = \frac{-1}{77}$$

$$B = 0,$$

so $y_p = -\frac{1}{77}\cos(9x)$

7. Find the general solution of the system

$\mathbf{X}_1, \mathbf{X}_2$

$$\mathbf{x}' = \begin{pmatrix} -3 & -4 \\ 1 & 1 \end{pmatrix} \mathbf{x}.$$

A. $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} te^{t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{t} \right],$

B. $c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} -2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} \cancel{1} \\ 0 \end{pmatrix} e^{-t} \right],$

C. $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} \right],$

D. $\boxed{c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} -2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} \right],}$

E. $c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} -2 \\ 1 \end{pmatrix} te^{t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{t} \right].$

eigenvalues $|A - \lambda I|$

$$\begin{vmatrix} -3-\lambda & -4 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)(-3-\lambda) + 4$$

$$= \lambda^2 + 2\lambda - 3 + 4$$

$$= \lambda^2 + 2\lambda + 1$$

$$= (\lambda+1)^2 = 0, \text{ roots } \lambda = -1, -1$$

$\lambda = -1$ alg. multiplicity 2

Need eigenvectors, i.e.

compute $\mathcal{E}_{-1} = \text{Null}(\underline{A} - (-1)\underline{I}) \Rightarrow \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow \left[\begin{array}{cc|c} -2 & -4 & 0 \\ 1 & 2 & 0 \end{array}\right] \begin{array}{l} \leftarrow -2a - 4b = 0 \Rightarrow a + 2b = 0 \\ \leftarrow \underline{a + 2b = 0} \Rightarrow \underline{a = -2b} \end{array}$$

eigenvectors are $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -2b \\ b \end{bmatrix}$, pick $b = 1$, $\underline{v_1} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

So $\dim(\mathcal{E}_{-1}) = 1$ $\qquad \mathcal{E}_{-1} = \left\{ b \begin{bmatrix} -2 \\ 1 \end{bmatrix} : b \in \mathbb{R} \right\}$

$$= \text{span}\left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\} \qquad \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

defect $= (\text{alg. mult } \lambda = -1) - (\dim(\mathcal{E}_{-1}))$

$$= 2 - 1 = 1$$

So need $\underline{v_2}$ a generalized eigenvector such that

$$\begin{cases} (A - (-1)I)\, \underline{v_1} = 0 \\ (A - (-1)I)\, \underline{v_2} = \underline{v_1} \end{cases} \Rightarrow (A - (-1)I)^2 \underline{v_2} = 0$$

$$(\underline{A} - (-1)I)\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} \overset{1}{\cancel{-2}} & \overset{2}{\cancel{-4}} & \overset{1}{\cancel{-2}} \\ 1 & 2 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 0 \end{array}\right] \leftarrow \begin{array}{l} a + 2b = 1 \\ a = 1 - 2b \end{array}$$

choices for $\underline{v_2}$ are $\begin{bmatrix} 1-2b \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

b free

pick $\underline{v_2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Solutions $\underline{X}_1 = e^{\lambda t}\underline{v_1} = e^{-t}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\underline{X}_2 = e^{\lambda t}(t\underline{v_1} + \underline{v_2}) = e^{-t}\left(t\begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$

general solution is $c_1\underline{X}_1 + c_2\underline{X}_2$

$$c_1 e^{-t}\begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1-2t \\ t \end{bmatrix}$$

$(r^2+1)^2 = 0$ $\qquad$ $\underline{r^2+1=0}$ has root pair $r = \pm i$

$(r^2+1)^2 = 0$ $\qquad$ has root pair $r = \pm i$, with mult. 2

11. Let $y(x)$ be the solution of the initial value problem

$$\underline{y'' + y = x,}$$
$$y(0) = 1, \quad y'(0) = 2.$$

Then $y(\pi)$ is equal to

A. $y(\pi) = \pi$

B. $y(\pi) = 2\pi$

C. $y(\pi) = \pi - 1$

D. $y(\pi) = 2\pi + 1$

E. $y(\pi) = \pi + 1$

need $y = y_c + y_p$

account for $f(x) = x \neq 0$

solution for homog. eq. $y'' + y = 0$

$y'' + y = 0 \rightarrow r^2 + 1 \quad r = \pm i$

$\underline{y_c = c_1 \cos(x) + c_2 \sin(x)}$

Step 2 : trial function

$y_p = A + Bx$

$$\left.\begin{aligned} y_p &= A + Bx \\ y_p' &= B \\ y_p'' &= 0 \end{aligned}\right\} \quad y_p'' + y_p = \underline{A + Bx} = \underline{x}$$

$A = 0 \quad B = 1.$

so $\underline{y_p = x}$

so general solution is

$y = y_c + y_p = c_1 \cos(x) + c_2 \sin(x) + x$

$y(0) = 1$

$y'(0) = 2$

$1 = y(0) = c_1 \longrightarrow \underline{c_1 = 1}$

$y' = -\cancel{c_1}^{1} \sin(x) + c_2 \cos(x) + 1$

$2 = y'(0) = c_2 + 1 \Longrightarrow \underline{c_2 = 1.}$

particular solution is

$y = \cos(x) + \sin(x) + x$

$y(\pi) = -1 + 0 + \pi = \pi - 1$

9. Let A_{ij} be the cofactor of the element a_{ij} in the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with $\det(A) = 5$. The value of the expression $a_{11}A_{11} + a_{12}A_{12} + a_{21}A_{21} + a_{22}A_{22}$ then is equal to

A. 0

B. 5

C. $\boxed{10}$

D. 15

E. undetermined by the information given above

i,j – cofactor of $\underline{A}$ is $(-1)^{i+j} M_{ij}$, where M_{ij} is the determinant of the matrix you get by deleting row i, column j of $\underline{A}$

$\det(\underline{A})$ can be computed by going along any row/column.

$$\underbrace{\boxed{a_{11}C_{11} + a_{12}C_{12}}}_{\text{determinant computed along first row}} + \underbrace{\boxed{a_{21}C_{21} + a_{22}C_{22}}}_{\det(A) \text{ along 2nd row}}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad + a_{11}C_{11} + a_{12}C_{12}$$

$$\underbrace{\boxed{a_{11}C_{11} + a_{21}C_{21}}}_{\det(A) \text{ along column 1}} + \underbrace{\boxed{a_{12}C_{12} + a_{22}C_{22}}}_{\det(A) \text{ along Col 2}}$$

$$= \det(A) + \det(A)$$

$$= 5 + 5 = 10$$

Compute $\det\left(\begin{bmatrix} 1 & 2 & 2 \\ -1 & 4 & 1 \\ 0 & 0 & 3 \end{bmatrix}\right)$

along column 1 : $a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

$$= C_{11} + (-1)\,C_{21} + 0 \cdot C_{31}$$

$$= (-1)^{1+1} M_{11} + (-1)\cdot(-1)^{2+1} M_{21} + 0\cdot(-1)^{3+1} M_{31}$$

(signs follow $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$)

$$= M_{11} + M_{21}$$

$$= \begin{vmatrix} 4 & 1 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 0 & 3 \end{vmatrix} = 12 + 6 = 18$$

along row 3 : $\cancel{a_{31}}^{\,0} C_{31} + \cancel{a_{32}}^{\,0} C_{32} + a_{33}C_{33}$

$$+\ 3\cdot(+)\begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix}$$

$$= 3\cdot(4+2) = 18$$

17. Let $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ satisfy

$$X'(t) = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix} X(t), \quad X(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

The limit $S = \lim_{t\to\infty} \frac{x_2'(t)}{x_1'(t)}$, which is the slope of the curve $X(t)$ as $t \to \infty$, is equal to

A. $S = -3$

B. $S = \frac{1}{2}$

C. $S = -\frac{1}{3}$

D. $S = -\frac{1}{2}$

E. $S = 3$

Gen solution for

$$\underline{X}' = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix} \underline{X}$$

eigenvalues $\rightarrow |\underline{A} - \lambda I| = \begin{vmatrix} 1-\lambda & 6 \\ 1 & 2-\lambda \end{vmatrix}$

$= (1-\lambda)(2-\lambda) - 6$

$= \lambda^2 - 3\lambda + 2 - 6$

$= \lambda^2 - 3\lambda - 4$

$= (\lambda-4)(\lambda+1) = 0 \implies \underline{\lambda = 4, \ \lambda = -1.}$

$\underline{\lambda = -1}$ Compute $\text{Null}(A - (-1)I)$

$$\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} \overset{1}{\cancel{2}} & \overset{3}{\cancel{6}} & 0 \\ 1 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} a + 3b = 0 \\ a = -3b \end{array}$$

vectors are $\begin{bmatrix} -3b \\ b \end{bmatrix}$, pick $b = 1$, $\underline{v_1} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

$\underline{\lambda = 4}$ $\text{Null}(A - 4I) \rightarrow \left[\begin{array}{cc|c} \overset{1}{\cancel{-3}} & \overset{-2}{\cancel{6}} & 0 \\ \underset{0}{\cancel{1}} & \underset{0}{\cancel{-2}} & 0 \end{array}\right] \leftarrow \begin{array}{l} a - 2b = 0 \\ a = 2b \end{array}$

vectors are $\begin{bmatrix} 2b \\ b \end{bmatrix} \Rightarrow$ pick $\underline{v_2} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

general solution $\underline{X} = c_1\left(e^{-t}\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right) + c_2\left(e^{4t}\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$

$\underline{X}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3c_1 + 2c_2 \\ c_1 + c_2 \end{bmatrix}$

$$\left.\begin{array}{l} -3c_1 + 2c_2 = -1 \\ c_1 + c_2 = 2 \end{array}\right\} \Rightarrow c_1 = c_2 = 1$$

particular solution is

$$\underline{X} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -3e^{-t} \\ e^{-t} \end{bmatrix} + \begin{bmatrix} 2e^{4t} \\ e^{4t} \end{bmatrix}$$

$$\begin{cases} x_1(t) = 2e^{4t} - 3e^{-t} \\ x_2(t) = e^{4t} + e^{-t} \end{cases}$$

want $\displaystyle \lim_{t \to \infty} \frac{x_2'(t)}{x_1'(t)} = \lim_{t \to \infty} \frac{4e^{4t} - e^{-t}}{8e^{4t} + 3e^{-t}}$

as $t \to \infty$, $e^{-t} \approx 0$, so we roughly have

$$\approx \frac{4e^{4t}}{8e^{4t}} = \frac{1}{2}$$

15. Let A be a 2×2 matrix whose entries are real numbers. If $\underline{\lambda = 2 + 3i}$ is a complex eigenvalue of A with corresponding complex eigenvector $\mathbf{w} = \begin{bmatrix} 1-i \\ 4 \end{bmatrix}$, then the general solution to $\mathbf{x}' = A\,\mathbf{x}$ is:

A. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t + \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t - \cos 3t \\ 4\sin 3t \end{bmatrix}$

B. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t - \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t - \cos 3t \\ 4\sin 3t \end{bmatrix}$

C. $\mathbf{x} = C_1\, e^{3t} \begin{bmatrix} \cos 2t + \sin 2t \\ 4\cos 2t \end{bmatrix} + C_2\, e^{3t} \begin{bmatrix} \sin 2t - \cos 2t \\ 4\sin 2t \end{bmatrix}$

D. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t + \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t + \cos 3t \\ 4\sin 3t \end{bmatrix}$

E. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t + \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t - \cos 3t \\ -4\sin 3t \end{bmatrix}$

don't know $\underline{A}$, but is 2x2. $\lambda = 2 \pm 3i$

If $\lambda = 2+3i$ is an e-value, then $\bar{\lambda} = 2-3i$ is also an eigenvalue.

Since $\underline{w} = \begin{bmatrix} 1-i \\ 4 \end{bmatrix}$ is an e-vector for $\lambda = 2+3i$, then

$\bar{\underline{w}} = \begin{bmatrix} 1+i \\ 4 \end{bmatrix}$ is an eigenvector for $\bar{\lambda} = 2-3i$.

$c_1 e^{\lambda_1 t} \underline{w} + c_2 e^{\lambda_2 t} \bar{\underline{w}}$, but we can make two solutions

$\mathbf{X}_1 = \mathrm{Re}(e^{\lambda t} \underline{w})$, $\mathbf{X}_2 = \mathrm{Im}(e^{\lambda t} \underline{w})$

(remember how "post-conversion" from e^{ix}, e^{-ix}, we had basis solutions $\cos(x)$, which $= \mathrm{Re}(e^{ix})$
$\sin(x)$, which $= \mathrm{Im}(e^{ix})$)

(using $e^{ix} = \cos(x) + i\sin(x)$, ... "basis solutions"
$y_1 = \cos(x)$
$y_2 = \sin(x)$)

Option 1: $\quad \mathbf{X}_1 = e^{(2+3i)x}\begin{bmatrix}1-i\\4\end{bmatrix}$

$$\mathbf{X}_2 = e^{(2-3i)x}\begin{bmatrix}1+i\\4\end{bmatrix}$$

$$\mathbf{X} = c_1\mathbf{X}_1 + c_2\mathbf{X}_2 = c_1 e^{2x}e^{3ix}\begin{bmatrix}1-i\\4\end{bmatrix} + c_2 e^{2x}e^{-3ix}\begin{bmatrix}1+i\\4\end{bmatrix}$$

and now...

$$c_1 e^{2x}\left(\cos(3x)+i\sin(3x)\right)\begin{bmatrix}1-i\\4\end{bmatrix} + \cdots$$

then collect / separate all real, imaginary pieces.

too messy...

Option 2 $\quad$ Take $\mathbf{X}_1 = \mathrm{Re}\left(e^{(2+3i)x}\begin{bmatrix}1-i\\4\end{bmatrix}\right)$

$$\mathbf{X}_2 = \mathrm{Im}\left(e^{(2+3i)x}\begin{bmatrix}1-i\\4\end{bmatrix}\right)$$

$$e^{2x}e^{3ix}\begin{bmatrix}1-i\\4\end{bmatrix} = e^{2x}\left(\cos(3x)+i\sin(3x)\right)\left(\begin{bmatrix}1\\4\end{bmatrix}+i\begin{bmatrix}-1\\0\end{bmatrix}\right)$$

$$e^{2x}\left(\cos(3x)\begin{bmatrix}1\\4\end{bmatrix} - \sin(3x)\begin{bmatrix}-1\\0\end{bmatrix}\right)$$

$$+ e^{2x}\left(i\sin(3x)\begin{bmatrix}1\\4\end{bmatrix} + i\cos(3x)\begin{bmatrix}-1\\0\end{bmatrix}\right)$$

$$= \boxed{e^{2x}\begin{bmatrix}\cos(3x)+\sin(3x)\\4\cos(3x)+0\end{bmatrix}} \qquad \mathbf{X}_1 = \text{Real part}$$

$$+ i\,\boxed{e^{2x}\begin{bmatrix}\sin(3x)-\cos(3x)\\4\sin(3x)\end{bmatrix}}$$

$\mathbf{X}_2$ = imaginary part

$$\underline{X}_1(t) = e^{2t}\begin{bmatrix} \cos(3t)+\sin(3t) \\ 4\cos(3t) \end{bmatrix}$$

$$\underline{X}_2(t) = e^{2t}\begin{bmatrix} \sin(3t)-\cos(3t) \\ 4\sin(3t) \end{bmatrix}$$

Finally! general solution for $\underline{X}' = A\underline{X}$ is

$$\underline{X} = c_1\underline{X} + c_2\underline{X}_2$$

$$= c_1 e^{2t}\begin{bmatrix} \cos(3t)+\sin(3t) \\ 4\cos(3t) \end{bmatrix} + c_2 e^{2t}\begin{bmatrix} \sin(3t)-\cos(3t) \\ 4\sin(3t) \end{bmatrix}$$